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THE MATHEMATICS TEACHER

VOLUME XVIII MAY, 1925 NUMBER 5

# MATHEMATICS IN THE JUNIOR HIGH SCHOOL

By JOHN R. CLARK,
The Lincoln School of Teachers College

## CONFERENCE I[1]

### What Mathematics to Teach in the Junior High School

*Mr. Jewell:* Two weeks from to-day, as you know, we plan to open the first junior high school in our city. Many of us have been asking ourselves in what respects the kind of education that our pupils will receive, or should receive, in this new school may be expected to differ from that which they have been receiving in our conventional type of school. While we are aware that there is developing in this country a *new education*, so to speak, few of us have any definite convictions about its general aims and philosophy. Still fewer of us feel that we know exactly what should be taught in any subject in the junior high school, even in mathematics, the subject in which we are particularly interested. Realizing that we should be more familiar with the ideals of this new education, the Board of Education has asked us to come together for a series of conferences for the purpose of formulating a course of study in mathematics. We want to enter into our new work with as much insight and understanding as possible. I feel that too many cities have adopted the junior high in name only. They have not appreciated its real function.

To help us think through the problems of making a course of study in mathematics for our new school, we have arranged to have certain of our friends from State College meet with us. Dr. Henderson, Dr. Jones and Dr. Tompkins certainly need no introduction to this group.

*Mr. Smith:* I'm not sure, Mr. Jewell, that we members of the Board, ever really understood just what you mean by the

[1] The characters in these conferences are:
Mr. Jewell, principal-elect of the new Roosevelt Junior High School.
Dr. Henderson, professor mathematics in State College;
Dr. Jones, professor of education in State College;
Dr. Tompkins, professor of psychology in State College;
The Misses Scott, Brown, Murphy, Ryan, O'Rourke and Davidson, and Messrs. Hardin and Hughes, recently appointed teachers of mathematics in the Roosevelt Junior High School;
Mr. Smith, a realtor, and a member of the Board of Education.

*new education.* We had enough confidence in you to accept your recommendation to adopt this new plan of organization, and besides, our neighboring cities are adopting it. But I should like to know just what you really mean by the *new education.*

*Mr. Jewell:* As I understand it, this idea was developed by certain distinguished American educators, including such men as Briggs, Davis, Inglis, Judd, Koos, and of course, our own Dr. Jones. Let's have Dr. Jones answer your question.

*Dr. Jones:* The new education is a term we apply to the aggregate of modern ideas about the meaning of education and the nature of the educative process. It implies, for example, that the purpose of education is to promote the good life. This means that health, ethical character, citizenship, industry, worthy uses of leisure, command of the fundamental processes, and the like, are to be basic in all our thinking about teaching. The term implies, also, that pupils learn more readily and more permanently, when they appreciate the significance of what they are learning. It suggests, too, that pupils develop more rapidly when they are actually engaged in investigation, in problem solving than when they are required to memorize a great many assigned tasks later to be reproduced for the teacher.

The old education had too little respect for the pupil as an individual,—for his needs, interests, and capacities. It cared little whether the subject appealed to the learner; whether it was a significant part of our social heritage, or whether the pupil would ever use it. The old education certainly made serious blunders in the way it sought to teach the subject in which you people are most interested.

*Mr. Smith:* What you say seems very sensible. But won't you be a little more specific in your explanation. Perhaps you could give some illustrations.

*Dr. Jones:* I'm very sorry for being so vague. To be more specific, there are five important aspects of the new education that we should keep in mind.

In the first place, we have recently begun to analyze social needs. Studies have been made to ascertain the specific kinds of mathematics that are needed in every day life. Much to our chagrin, there is a wide gap between the kind of mathematics that we teach and the kind that is most generally used in modern life. In other words, we have realized that there are many things

that pupils should know that are not taught in the schools, and many things that are taught in the schools that are of comparatively little value.

In the second place, the new education rests upon a sounder psychological basis. We now know a great deal about the ways in which children learn. Children's minds work in very definite ways. There are good ways and poor ways of teaching. Our knowledge of modern psychology enables us to save much time and effort through improved methods of teaching. We shall certainly wish to discuss this point at great length when we advance a little further with the formulation of our course of study. I'm sure that Dr. Tompkins will help us a great deal on this topic.

Again, we have only recently appreciated the extent to which pupils differ in their ability to learn. We now realize that it is folly to expect the same results from the same effort on the part of different pupils. I dare say that you, Miss Scott, have had pupils in your classes who never could learn to solve a quadratic equation.

*Miss Scott:* Worse than that. I've had pupils in algebra classes who couldn't add $\frac{1}{2}$ and $\frac{1}{3}$ and get anything but $\frac{2}{5}$.

*Dr. Jones:* I regard the use of educational measurements as the fourth important factor in the new education. We are rapidly becoming able to measure by means of tests what a pupil has learned by this or that method in this or that length of time. We shall presently be able to study the efficiency of various methods of building up knowledge or power in the minds of our youth in the same scientific way in which we study the efficiency of the gas engine. I assume that we shall be concerned to make the best use of the various standard tests as aids in our teaching.

*Miss O'Rourke:* I have made a special study of the ways in which these tests are helpful to us. In fact I have a paper on that topic.

*Mr. Jewell:* That's fine. We'll have a place for that, Miss O'Rourke. Did you not have another point, Dr. Jones?

*Dr. Jones:* I was just coming to that. It applies specifically to the junior high school, and is accepted by practically all of the eminent educationists. It is: In the junior high school grades, we believe that the pupil should explore the significant fields of knowledge so that he may secure some notion of their nature and

thereby discover his aptitudes and interests in these fields. The approach to each of these important branches of knowledge should be by way of general introductory courses, containing the most worthwhile elements in each of the major branches. The larger more fundamental aspects of the courses, not the details, should be emphasized. This point of view, as you see, is quite opposed to the older, more prevalent notion.

*Dr. Henderson:* I am particularly interested in the last suggestion made by Dr. Jones. Am I correct in thinking that you recommend a general introductory course in mathematics, a course in which arithmetic, algebra, geometry and possibly trigonometry will be *sampled*, to extend throughout grades seven, eight and nine?

*Dr. Jones:* Quite right. The fundamental philosophy of junior high school education states that these general introductory, overview courses more effectively meet pupils' needs, whether they be in their pursuit of a livelihood, culture or happiness. What do you think of the idea, Dr. Henderson?

*Dr. Henderson:* I never approve or disapprove a proposition until I have thought it through most carefully. But I am inclined to think that I agree with your general principles. I am certain that we need a change from our present practice of teaching only arithmetic in the first eight grades. We waste a great deal of time this way. And I am very certain that something is radically wrong with our first year algebra. It means very, very little to many of those who take it. You would be shocked if I were to tell you how little algebra pupils know when they enter my college classes. They seem to have learned absolutely no mathematical principles. I should emphasize power to reason and do precise thinking rather than so much manipulation.

*Mr. Smith:* I have been very much benefited by this discussion. I had hoped, however, to learn more of the particulars about the course in mathematics, as you plan to have it this year. Shouldn't mathematics be more practical—more usable? How do you think the course will differ from the older courses, Mr. Jewell?

*Mr. Jewell:* From my twenty years experience in teaching mathematics, from my study of the School Curriculum and from what I know about psychology, I think our new course should have the following characteristics:

It should be practical. I think the course should be selected to prepare for actual or highly probable social needs. The most useful skills, concepts and attitudes to be derived from the entire field of elementary mathematics should constitute the new course.

It should be simplified. Courses in mathematics have been undergoing a continuous process of simplification for half a century. The more complicated and less practical elements have gradually been dropped. This movement has probably sufficiently reduced the difficulty of courses for the "above average" pupils. But further simplification for the "average" and "below average" pupils seems to me to be imperative. This movement will likely continue until the courses are within the comprehension of three-fourths, or more, of the pupils who take them. Differentiated courses, in which the degree of difficulty and the capacity of the pupil are not incommensurable, should be developed.

The course should be extensive rather than intensive. Much emphasis is being placed upon the importance of offering pupils in these grades a general survey of the major fields of knowledge. To provide the opportunity for an "over view" or exploration of the subject of mathematics, it is necessary to choose materials from algebra, geometry, trigonometry and arithmetic. Within sensible restrictions an "exposure" to a wide range of mathematical subjects and methods should replace the traditional intensive drill upon more limited fields of mathematics, such as we have seen in our own classes.

Emphasis should be upon power rather than technique. Mathematicians have generally insisted, as did Dr. Henderson, that power to reason and an appreciation of precise thinking constitute the more permanent outcomes from the study of mathematics. In the school, however, the nature of both the materials taught and the methods employed in their teaching have inhibited these outcomes. Formal, uncritical learning has replaced the enquiring, reflective, research spirit that should characterize the teaching of secondary mathematics. It is hardly necessary to state that technique has its place as a necessary tool, because it has so greatly overshadowed attention to discovery, to generalization, to *mathematical* thinking. Tomorrow we shall discuss the big, major, aims of mathematics. I am sure that Mr. Smith will then feel that we have our "feet on the ground."

## Conference II

### What Are the Larger Objectives of Mathematics in in Junior High Schools?

*Mr. Jewell:* What are the valid aims or objectives of mathematics in junior high schools? Some may think this a needless question. But isn't it true that the kind of mathematics that we shall decide to teach will depend very largely upon our reasons for teaching the subject,—upon the objectives or aims that we accept as valid? What aspects of the whole field of mathematics should be emphasized in junior high schools?

*Dr. Jones:* I believe that we should teach the mathematics that is most useful in modern life. For example, any good citizen should know something about common every day business practice. By this I mean that he should understand the meaning and uses of check, notes, receipts, interest, discount, taxes, and the like. Here we have need for two kinds of mathematics—computation and business practice. Accordingly, I would propose two large objectives, to be kept in mind when we come to select the details of the course.

*Mr. Hardin:* I certainly agree with the first point made by Dr. Jones. Everybody needs to know how to compute. But I would go a bit further, and say that they should not only know how to compute but that they should actually compute with 100 per cent accuracy. I think it is high time for us to take a stand against the terrible inaccuracy that you see in the average class room. I'm going to *demand* accuracy in computation, if it is the only thing I do this year.

*Miss Murphy:* I should think that pupils would have sufficient skill in computation when they enter junior high school. What have they been taught in arithmetic in the elementary school? Don't they have sufficient skill? I once heard a professor of education say that pupils could learn all the arithmetic they would ever need during the first six grades.

*Mr. Jewell:* The fact is, Miss Murphy, that pupils do not acquire dependable habits of computation in the first six grades. And since they do not, we surely need to consider this as one of our big problems. Without attempting now to decide just how much skill in computation is needed, let us agree that our first

responsibility to our pupils is to see to it that they develop a facility in computing. Dr. Tompkins, have you a suggestion?

*Dr. Tompkins:* I have often been amused to hear teachers of mathematics claim that mathematics develops in the pupil ideals of precision, and accuracy when they tolerate a degree of inaccuracy in computation which results on the average in one error out of four computations! To be serious, I am in hearty sympathy with your emphasis upon greater accuracy. Pupils should be taught that it is not only futile, but altogether wrong to leave a computation until, by checking, they are morally certain of the accuracy of their results.

*Miss Brown:* Wouldn't it be in order to ask Dr. Tompkins to tell us how to secure greater accuracy?

*Mr. Jewell:* That's very important, but I think we should postpone a discussion of it till a later conference. Is there anything more to be said about the uses of mathematics in connection with business practice? You recall that Dr. Jones considers an understanding of common business practice and the ability to solve every day problems connected with it, very important in modern life. Shall we consider the teaching of business practice a really major objective in our course?

*Mr. Hughes:* I never thought that I was teaching real mathematics when I taught such topics as stocks and bonds, mortgages, insurance, investments. They seem to me to be primarily informational, more like social studies. Computation plays a small part in an understanding of these aspects of business practice. Should we not leave these things out of our course, and put in some real mathematics—some intuitive geometry and some algebra?

*Dr. Jones:* I protest against the implication of the last remarks. If the primary purpose of teaching mathematics is to help people to think *quantitatively* about the problems of the home, the farm, the store, the local government, and the like, then it certainly is important to teach our youth the meaning and uses of such concepts as budgets, simple bookkeeping, something about investments and insurance, and certainly an appreciation of the way money or savings grow when touched by the magic of compound interest! You are quite right, Mr. Hughes, in suggesting that this is not *pure mathematics.* But it is mathematics at work in the solution of personal, home and community

problems. Now, of course, it may be that business practice should be taught in connection with some subject other than mathematics. But we have to face facts as they are. The teaching of business practice has been assigned to the department of mathematics and it is our responsibility to accept it.

*Miss Davidson:* I have just taken a course on the Reconstruction of the Curriculum at the University with Professor Johnson in which this very issue was raised. Professor Johnson believes thoroughly in teaching pupils the uses of mathematics in the home, the store, the shop, in the bank,—in business practice, as he calls it. He thinks that we teachers of mathematics often fail, however, to emphasize the informational and meaning aspects. According to his theory, it is a mistake to spend only a few minutes teaching the meaning and use of taxes, let us say, and to spend a great deal of time in figuring examples such as "compute the tax paid on $21,600 worth of property when the tax rate is $2.50 per $100." I believe that we have had the wrong idea about this whole problem. I agree with Dr. Jones that whether business practice is mathematics or not, it is most important when properly taught. It teaches worth-while facts, more so than does algebra or geometry.

*Mr. Jewell:* Then we shall accept as our second major objective the teaching of common business practice. I was practically certain that we would decide to do so. But how are we to decide upon the more specific facts about business practice to be included in our course? Perhaps we should also delay the discussion of this problem until a later conference. In the meantime we will have an opportunity to consult any researches that may have been made in this field.

Just one more word on the teaching problem. Isn't the real crux of the whole problem that of giving pupils first-hand experiences with the various forms and methods of business practice? Schools that teach banking practice through the operation of a school bank, or schools that organize corporations to conduct school enterprises, certainly make these aspects of business practice meaningful and significant. We should, of course, in the absence of this first-hand experience use vivid descriptions.

*Dr. Henderson:* I hope that we are not going to limit our junior high school mathematics to the development of dependable habits of computation and to an understanding of the

simple, generally used, facts and processes of business practice. Let's go a little further with the analysis of the kinds of mathematical problems that people find it advantageous to be able to solve. We shall soon meet the age old question that people ask about material objects: *What is their size? What is their shape? What is their position or location?* Are we not going to teach pupils these notions about size, shape and position? Intuitive geometry gives the answers to these questions. It teaches pupils to measure length, area and volume; to classify various geometric forms, to construct many geometrical figures and designs, and to appreciate the wide role of geometry in art, furniture, clothing, architecture and nature. I regard this as one of the most educative subjects in the whole field of mathematics. It is stressed, as you know, in practically all European schools in the years corresponding to our junior high school grades. And of course, I need not say to you that intuitive geometry is an ideal preparation for demonstrative geometry. It is practically a prerequisite for the study of demonstrative geometry.

*Miss Ryan:* I have never clearly understood just what intuitive geometry really is. There may be other teachers here, too, who are not any too clear about it. Is intuitive geometry the same as the mensuration that we studied in the old arithmetics? Is it not just a new name from an old idea? As you may suspect, I've been teaching long enough to have seen one fad after another come and go. I don't believe in panaceas.

*Dr. Henderson:* Intuitive geometry is a study of the properties of space, by investigation and experiment. It is a laboratory subject. Pupils measure, draw inferences, check up these inferences. They learn to use the methods of science. In these respects, intuitive geometry is unlike both the mensuration of the older arithmetics and the demonstrative or logical geometry of Euclid.

*Mr. Jewell:* I infer from this discussion that there are different ways to learn a fact, particularly some geometrical fact. Dr. Tompkins acts as if he wished to speak on this subject.

*Dr. Tompkins:* I wanted to call attention to three ways of learning the fact that the sum of the angles of a triangle is one hundred eighty degrees. You might teach this fact *experimentally* by having your pupils measure the angles of a number of triangles, find the sum of the angles of each triangle, compare

these sums, compute their average, and draw the conclusion that the sum must be 180 degrees. This experimental method, when based upon refined observation and measurement, leads to strong conviction. It is the method by which pupils arrive at a great many generalizations about spatial relations.

There is a second method by which you may convince your pupils of this fact. To illustrate, you may turn a pencil through the angles of a triangle, as I shall indicate by this figure. Please note that in turning through the three angles the pencil has reversed its direction, and has, therefore, turned through 180 degrees. You will agree that this method leads to strong conviction. A pupil will see *intuitively* that the pencil in turning through the three angles, has turned through a straight angle. This is a very good illustration of intuitive geometry. Most of us first learned this fact by another method—that of demonstration or logical proof. By the use of axioms, postulates, definitions or previously proved facts, we proved that this was true.

*Miss Scott:* Which of these methods do you recommend for junior high school pupils?

*Dr. Tompkins:* The logical or demonstrative method should certainly follow the others. Even though measurement and intuition are fallible, and demonstration is infallible, I should certainly use them first. If time permits, I should use all three. The typical eleven or twelve year old pupil revels in the explorations of space relations that are opened up to him by the methods of intuitive geometry. The skilful teacher, of course, will gradually lead the pupil to be skeptical of his conclusions and thereby prepare him for the more certain method of establishing conclusions, the method of demonstration or proof.

*Miss O'Rourke:* I like this explanation of intuitive geometry. I fear that I have not given my pupils sufficient opportunity to investigate, to experiment. Apparently there is considerable educational value in the learning of the fact. Pupils not only learn the particular fact, but they also learn how to investigate or how to study.

*Dr. Jones:* I think we might as well admit that pupils soon forget many of the facts that we teach them. The more permanent values often are the attitudes and general notions that are developed. Intuitive geometry certainly is rich in the possibilities it offers for teaching pupils ways of discovering things for

themselves. The mathematicians are right in attributing great value to intuitive geometry. I never before appreciated its educational possibilities. I think that Dr. Henderson suggested the third major purpose of teaching mathematics when he said people need to determine the form, size and position of objects in space. Shall we agree, then, that the third major objective of mathematics in junior high is an understanding of the properties of common space forms and relations, together with the development of the ability to solve the more practical problems pertaining to these forms?

*Mr. Jewell:* All seem to be agreed. For the reason, then, that everybody needs to be familiar with the properties of common space forms, and for the further reason that pupils will not have learned these before entering junior high school, we have decided to emphasize the study of space relations or intuitive geometry, as we call it, in our course of study.

Are there other big aims or objectives that we should keep in in organizing our course of study?

*Dr. Henderson:* Well I should say we haven't even mentioned the most significant notion, as a mathematician sees it. The really great, fundamental, unifying notion in mathematics is that of relationship or law. Let me give a number of illustrations.

The speed at which an object falls toward the earth increases very rapidly during its descent. During the second second it falls many more feet than during the first second, and so on for subsequent seconds. There is a definite relation or law between the distance (in feet) and the time (in seconds). As you know, $d$ and $t$ are variables that change together according to the law $d = 16t^2$. Or take the relationship between weight and distance in the familiar teeter board. We want this relationship, $\frac{W_1}{W_2} = \frac{l_1}{l_2}$ to be full of meaning to pupils. I am asking that more attention be given to this notion of law,—the function concept, as it is often called.

*Mr. Hardin:* In my work in physics and chemistry I learned to appreciate the importance of being able to think in terms of mathematical law. We were constantly using the notion of direct or inverse proportionality. I think more attention should be given to this phase of mathematics.

*Miss Ryan:* All of us teach how to find the area of circles. Probably most of us have our pupils express the rule by the formula $A = \pi r^2$. But if we stop here, they get no adequate notion of the relationship between the two variables, $A$ and $r$. We should go a step farther, and bring out the idea that $A$ varies as the square of $r$, meaning that when $r$ is doubled, $A$ becomes 4 times as great, and so on; and that you can express the relation between $A$ and $r$ graphically or in the form of a set of tables from which you can read any value of one variable corresponding to a given value of the other. I should also wish my pupils to see that the law, $A = \pi r^2$, may be written in the proportion form, $\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$

*Miss Brown:* We aren't going to teach only such algebra as is required for success in using these mathematical laws? To do so would probably exclude a large part of the work with polynomials, factoring, special products, fractions and simultaneous equations. I shouldn't want to see these "old standbys" deserted.

*Dr. Henderson:* I should like to see us build a course in which this function concept is the central unifying principle. Let us choose only those elements of the conventional first year algebra that contribute to the development of this fundamental notion. Pupils should learn something about how these laws are discovered. They should associate the three methods of expressing relationships—the formula, tabular and graphic methods. They should learn how to derive new statements of laws. We certainly believe that they should solve many genuine problems involving these laws.

*Dr. Jones:* This discussion of the function idea has certainly been a revelation to me. I never even heard of it in the courses that I studied. I can see, however, that it is one of the most practical notions in mathematics. Besides, it has wide application in other branches of knowledge. Apparently it is one of those broad concepts that proves helpful outside the mathematics classroom.

*Mr. Jewell:* Then you wish to emphasize the notion of relationship between quantities that change together according to some mathematical law? For example, you wish pupils to appreciate such laws as (1) the value of a fraction varies directly as

its numerator and inversely as its denominator, (2) the volume of a gas is inversely proportional to the pressure to which it is subjected, (3) the horsepower of a gas engine varies as the number of its pistons, and the square of their area, (4) the volumes of similar solids are to each other as the cubes of their corresponding dimensions, (5) in a teeter board, weight is inversely proportional to lever-arm, and the like. You wish pupils to think of algebra as a living, dynamic thing. You believe that the practical, cultural and disciplinary values to be derived from this type of algebra exceed those that were obtained from the older courses in which formal manipulation of algebraic numbers were emphasized.

Let us agree, then, that the fourth fundamental purpose of teaching mathematics in junior high schools is to teach pupils to think in terms of mathematical law, to use its language or notation, and to develop some facility in applying (and discovering, wherever possible) these laws in the solution of problems.

We called this second conference, as you know, to discuss the larger aims that should be kept in mind in making our course of study. So far, four important objectives have been agreed upon. Are there not other major objectives?

*Miss Davidson:* I read an article in the MATHEMATICS TEACHER on *Geometry in the Junior High School* in which the author expressed the idea that we should teach some demonstrative geometry. He argued that since only a small per cent of our pupils study mathematics beyond the ninth grade, it is highly desirable to include at least an introduction to the idea of *proof* in the ninth grade. Everybody, according to the writer, should know what it means to *prove* a point—how to make an unassailable argument. There is another reason for considering this question. Junior high school courses, as you know, are supposed to survey or explore the major fields of knowledge. Certainly a pupil has not had an exposure to the whole field of elementary mathematics if he has not had a "look in" on Euclidean geometry. Should we not accept this as a fifth important aim of our course?

*Dr. Henderson:* Wouldn't it be better to omit the geometry? Junior high school pupils are very immature. Geometry was designed for grown-ups. It was a college study until comparatively recently. Moreover, pupils do not really appreciate the

meaning and nature of proof until they have spent three or four months on it. I don't believe we have this amount of time for geometry in the ninth grade. It may be feasable for very superior classes, but I should oppose it as a requirement.

*Mr. Jewell*: How many agree with Dr. Henderson? Practically all, you see. Then we shall, for the time being at least, confine our list of major objectives to the four already agreed upon.

In the next conference we shall report on the investigations that have been made to find out the kinds of mathematics that are most generally used by people today. We are adjourned.

## Conference III

### What to Teach in Junior High School Mathematics

*Mr. Jewell:* In order to realize the primary objectives that we selected in the last conference, we need to decide upon the elements of arithmetic, geometry and algebra that should be taught. What skills in computation should be developed? What items of information about business practice should be included? What should be the content of intuitive geometry? What elements of algebra should be emphasized? These are the questions that we must consider if we are to formulate a sound course of study. To throw light on these questions, I have asked certain members of the committee to summarize the investigations that bear on these problems. Miss Davidson will tell us, briefly, the results of the studies that relate to every-day arithmetic.

*Miss Davidson:* Professor G. M. Wilson made an extensive study of the 14,583 problems that 4,068 adults, from whom he collected information, actually solved in their every day affairs during a period of two weeks. About one-third of the problems required multiplication; roughly one-fifth involved addition; subtraction was used in about one-eighth of them; division in about one-tenth; fractions in one-tenth; less than $\frac{1}{20}$ of one per cent of the problems required decimals other than United States money; and only 3 of the 14,583 problems required square root.

*Dr. Jones:* How difficult were the operations that were used in these problems?

*Miss Davidson:* The table on the blackboard (shown below) shows the number of 'places' in the addends, subtrahends, multipliers and divisors.

| | 1P | 2P | 3P | 4P | 5P | 6P | Total No. of Problems |
|---|---|---|---|---|---|---|---|
| Addition | 79 | 2143 | 1655 | 467 | 66 | 6 | 4416 |
| Subtraction | 222 | 930 | 1101 | 504 | 69 | 7 | 2833 |
| Multiplication | 3705 | 2733 | 483 | 53 | 0 | 0 | 6974 |
| Division | 966 | 1058 | 310 | 102 | 1 | 0 | 2437 |

From this table we see that these problems involved only the simplest kind of computations. The ability to multiply a four place number by a three place number is probably adequate for 99% of the multiplications that the ordinary citizen needs to perform. Very few problems in division involved divisors with more than three places.

*Dr. Tompkins:* How complicated or involved types of computation with integers should we teach? I should like Dr. Henderson's opinion.

*Dr. Henderson:* We certainly should secure high mastery on the types of computation that are most frequently used in life. I think we should teach the more complicated types now and then so that pupils will not feel limited to these easier types. The more capable pupils, at least, derive a keen satisfaction in being able to apply a process to any number,—they like to generalize. Have you any information concerning the use of fractions?

*Miss Davidson:* Yes, Professor Wilson found that fractions occurred in 1974 of the problems. This table (given below) shows their frequency of occurrence.

| Fraction | Frequency | Fraction | Frequency | Fraction | Frequency |
|---|---|---|---|---|---|
| $\frac{1}{2}$ | 1289 | $\frac{2}{5}$ | 15 | $\frac{7}{10}$ | 8 |
| $\frac{3}{4}$ | 189 | $\frac{4}{5}$ | 13 | $\frac{1}{6}$ | 6 |
| $\frac{1}{4}$ | 159 | $\frac{1}{5}$ | 10 | $\frac{1}{12}$ | 5 |
| $\frac{1}{3}$ | 119 | $\frac{5}{8}$ | 8 | $\frac{3}{5}$ | 4 |
| $\frac{2}{3}$ | 54 | $\frac{1}{10}$ | 8 | $\frac{7}{8}$ | 4 |
| $\frac{3}{8}$ | 20 | $\frac{7}{10}$ | 8 | all others | 50 |

As you see, only the simplest fractions were used in these problems. In my opinion the ability to find the sum, difference, product or quotient of any two of the fractions in the table is

sufficient for the solutions of 98% of the problems that adults, excluding those in technical pursuits, ordinarily need to solve.

*Dr. Jones:* I suggest that we limit our teaching to these very simple fractions. Why teach pupils how to add eights and fifteenths, for example, when this skill is not needed in the solution of life's problems? The time required for this could better be spent on other things.

*Dr. Henderson:* The schools have already over-simplified their teaching of fractions. I want pupils to know some principles,—to understand the process of adding fractions. For example, I want them to appreciate such principles as

(1) only like fractions may be combined.

(2) a fraction may be changed into an equivalent fraction by multiplying, or dividing, its terms by the same number.

An understanding of these principles is the best insurance against the disaster of obtaining $\frac{5}{9}$ for $\frac{3}{4} + \frac{2}{5}$. Mathematics is a subject that is to be understood. I should favor teaching pupils how to add *any* two fractions.

*Miss Davidson:* You will be interested, Dr. Henderson, in the list of topics or process that were not used at all in the 14,583 problems. The list includes:

(1) Greatest common divisor and least common multiple beyond the power of inspection; (2) long, confusing problems in common fractions; (3) complex and compound fractions; (4) reductions in denominate numbers; (5) table of folding paper, surveyor table, tables of foreign money; (6) compound numbers, neither addition, subtraction, multiplication nor division; (7) longitude and time; (8) cases 2 and 3 in percentage; (9) compound interest; (10) annual interest; (11) exchange, neither domestic nor foreign; (12) true discount; (13) partnership with time; (14) ratio, beyond the ability of fractions to satisfy; (15) most of mensuration,—the trapezoid, trepezium, polygon, frustum, sphere; (16) cube root; and (17) the metric system.

Most of us teach a number of these topics. For example, the "cases" in percentage, compound interest, certain topics in mensuration, and the metric system. Should we eliminate these, and other similar topics, from our courses?

*Dr. Henderson:* May it not be true that many of the persons reporting *should have* used one or more of these topics? For example, should not a farmer each year compute the yield that he secures on his investment? If so, then "Case 2" in percentage is necessary. Or should he not be able to compute the value of a 2,142 pound load of wheat at $1.80 per bushel? If yes, he uses reduction of denominate numbers.

These inventories or surveys of social usage are of inestimable value in the determination of what *is*. They do not suggest what *should be*. Progress can come only through a preparation for what should be.

*Mr. Jewell:* The Wilson investigation suggests that people in general use a less difficult and complex type of arithmetic than the schools have been teaching. That adults *should* use so little arithmetic is seriously to be questioned. Wilson found no reference to investments in stocks, bonds, and mortgages; no problems concerned with mortgages, building and loan association; no instance of the use of compound interest. Students of society, however, have adequate evidence that these topics are met by adults, and that our intelligent understanding of them (which certainly involves some computation) would make for more effective living. We surmise that the Wilson study overlooked many uses of arithmetic of great importance to the men and women from whom the data were sought.

I think we may safely conclude that we should emphasize higher mastery of a less complicated type of computation.

Miss Brown will now summarize a study that was made for the purpose of finding out the minimum information that should be known about banking.

*Miss Brown:* In 1919 Miss Alice Camerer, of the University of Iowa, sent a questionnaire to a large number of bank employees to secure their judgment of the relative importance to the people of a community, of a large number of facts about banking. From the replies these facts were arranged according to their importance. The twenty most important facts were (1) How to write checks, (2) how (and why) to fill out a stub, (3) when checks should be cashed, (4) how to stop payment on a check, (5) how to indorse checks, (6) what to do if a check is lost, (7) how to indorse notes, (8) how to write a negotiable note, (9) how to indorse a check in full, (10) how to use a bank book, (11) how to make out a deposit slip, (12) how to compute interest, (13) importance and purpose of savings banks, (14) the importance and purpose of commercial deposit banks, (15) how to use promissory notes, (16) the responsibility of the maker of a note if it is lost, (17) the meaning and use of certified checks, (18) how to open a bank account, (19) when notes are void, and (20) knowledge of legal rates of interest.

I suggest that we use the results of this study in making our course in business practice.

*Mr. Jewell:* Most assuredly. It is unfortunate that we have no similar studies on the other important aspects of business practice. Let us hope that investigations will soon be made to determine the minimum information about insurance, taxes, stocks and bonds, mortgages, bookkeeping, and the like. Until such studies are made we shall have to use our best judgment.

We shall now have some reports on the uses and importance of the topics in algebra. Mr. Hughes will summarize the studies made by Rugg and Clark in their *Scientific Method in the Reconstruction of Ninth Grade Mathematics.*

*Mr. Hughes:* Rugg and Clark, in 1917, became interested in measuring pupils' mastery of the elements of first year Algebra. They soon became convinced that first year algebra, as it was then organized and taught, meant very little to the majority of the pupils who took, even to most of those who "passed" it. Naturally, they began to think about an improvement of the course. Two general principles or criteria for selecting the details of the course were agreed upon: the social criterion and the psychological criterion. The first criterion or guiding principle was that the course should emphasize the most generally used mathematical skills, concepts and attitudes. The psychological criterion required that the materials of the course be arranged and presented so that they may be learned most effectively. It emphasized the very great importance of *thinking,* as opposed to memoriter learning.

In order to determine the mathematics that is most useful in life today, they analyzed a number of text books in physics, chemistry, general science, to find out exactly what mathematical skills or notions were required for success in these subjects. They also analyzed texts on practical mathematics, and a great many manuals of instruction issued by corporation schools for the further training of their employees. Let me state a few of their findings.

The mathematical topics that are most frequently used in physics courses are evaluation, simple equations, ratio and proportion and arithmetic. In chemistry ratio and proportion, evaluation and arithmetic are the most important. In the practical courses, measurement is all important. Finding very little,

if any, use for the operations with polynomials, special products, factoring, and simultaneous equations, they conclude that a large part of the conventional first year algebra cannot be defended on the ground that it meets a definite human need of a reasonably large proportion of the pupils who take it.

*Miss Brown:* I should like to know more about the psychological criterion, and its implications.

*Dr. Tompkins:* If Mr. Hughes doesn't mind, perhaps I may answer this question. Let us assume that we have determined the most intrinsically valuable mathematics, the mathematics that is most generally needed in life to-day, including perhaps some of the notions of measurement, some work in graphs, probably a great deal about formulas, certainly many of the facts that we classified as intuitive geometry, and the like. Now the psychological criterion is our guide when we come to arrange or organize all of these materials into a course of study. We might teach them merely by telling and retelling or we might set up situations so that pupils would be helped to think their way through them. By the psychological criterion I understand that subject matter is to be presented so that it will be most effectively learned. To arrange the course psychologically, we would have to consider such questions as pupils' interests, the amount and distribution of drill periods, the difficulty of the material, the nature of problem solving, and the like. These ideas are all very significant to a psychologist. He believes that very different results may be secured from two different organizations of the same body of material. He also believes that the educational value of the material depends not only upon the material itself, but upon the way it is presented, the method by which it is learned. Very great progress has been made in the increase in our information concerning the way children learn. I trust this explanation has been clear, Miss Brown.

*Miss Brown:* I think I understand. I should like to know what kind of mathematics Rugg and Clark recommended for pupils who had completed arithmetic.

*Mr. Hughes:* They recommended a course in which the emphasis would be upon insight and power rather than upon manipulation of algebraic symbols. They would emphasize the construction of formulas, evaluation, measurement (direct and

indirect) practical computation, variation and proportion, and simple statistics. Much importance would be attached to the notion of relationship, or functionality. Drill would be limited to those skills that are essential to a mastery of the significant parts of the course.

*Mr. Jewell:* Thank you, Mr. Hughes, for the summary of the Rugg-Clark recommendations. Miss Davidson is going to report briefly on the Thorndike investigations in algebra.

*Miss Davidson:* Professor Edward L. Thorndike and a group of specialists in high school mathematics recently (1922) undertook to find out the uses of algebraic abilities and skills as a preparation for work in the sciences in colleges and secondary schools, the uses of algebra in other high school subjects, and the kinds of mathematics used in the various articles in the Encyclopedia Brittanica. The results of these and many other valuable studies are reported in their excellent book, *The Psychology of Algebra*, published by the Macmillan Company. Thorndike found that the most extensive need for algebra in the non-mathematical subjects of the high school is the ability to understand graphic representations. What algebra do you suppose he found most useful in courses in physics? A mastery of the formula, particularly the ability that Nunn calls 'changing the subject of the formula'; the ability to choose from a number of formulas the one that is appropriate for the particular problem in hand; skill in making substitutions in formulas; and ability to use proportions,—these are most essential for success in physics. And again in chemistry the ability to deal with proportions is most important. I was particularly interested in Thorndike's statement that "involved manipulation of polynomial expressions is not a justifiable way of using the high school student's time."

Summing up their recommendations for a course in algebra, the Thorndike group urge us to "extend the field of application of the construction of formulas as well as their evaluation" to develop the art of criticism as applied to graphs; and to emphasize the notion of mathematical law.

*Mr. Jewell:* In this conference we are reviewing the literature that bears on the mathematics of grades seven, eight and nine. Please keep in mind that we shall presently be obliged to formu-

late a course of study for our own school. There is one other investigation that should be reported, the one made by the National Committee on Mathematical Requirements. Miss Murphy will summarize it for us.

*Miss Murphy:* The National Committee endeavored to find out what topics in high school mathematics are of greatest worth to college students. Since many college students do not continue the study of mathematics, but do pursue courses in the physical or social sciences, the content of high school courses should be selected with some reference to the mathematical prerequisites for success in these courses. In order to secure expert judgment concerning the relative value of a wide variety of mathematical subject matter, a number of well-known college professors were asked to assign to each of a large number of mathematical topics their estimate of its value as preparation for the elementary courses in their respective subjects.

The topics which were considered of very high value were simple formulas—their meaning and use, linear and quadratic functions and variation, numerical trigonometry, the use of logarithms and other topics relating to numerical computation, and statistics. It is interesting to note that these topics, formerly not included in the courses of study in grades seven, eight and nine, were considered of greater value than progressions, the binomial theorem, the theory of exponents, simultaneous quadratic equations and literal equations.

*Dr. Jones:* These studies suggest to me that the mathematics that is most useful in later high school or college courses is really not much different from the mathematics that most adequately meets the needs of junior high school pupils. I have always believed that the curriculum that best fits the present needs, interests, and capacities of boys and girls is quite similar to, if not identical with, the curriculum that best prepares for their future needs. This is, in fact, one of the tenets of the new education. It is very gratifying to find that your researches have practically proven this belief of mine.

*Mr. Jewell:* In any conference on What Mathematics to Teach in Junior High Schools, the recommendations of the National Committee on Mathematic Requirements should certainly receive serious attention. The committee, through its monumental

report[1] *The Reorganization of Mathematics in Secondary Education,* is destined to exert a far reaching influence in improving the teaching of mathematics in this country. Its recommendations, as you know, are based upon sound scholarship, the results of successful teaching experience, and the consensus of opinions of thousands of forward looking teachers. I have asked Dr. Henderson to present the gist of the recommendations that relate specifically to the course of study in grades seven, eight and nine.

*Dr. Henderson:* Accepting the general philosophy of the junior high school, the Committee desires "to give the pupil as broad an outlook over the various fields of mathematics as is consistent with sound scholarship." It wishes to provide the pupil with an opportunity to find himself, to test his abilities and aptitudes, and to secure information and experience which will help him to choose wisely his later courses and ultimately his life work. These are desiderata of the new education that we discussed in the first conference.

The Committee would spend less time on arithmetic; it would secure greater mastery of the commonly used skills. Reading from the Report:

> The amount of time devoted to arithmetic as a distinct subject should be greatly reduced from what is at present customary. This does not mean a lessening of emphasis on drill in arithmetic processes for the purpose of securing accuracy and speed. The need for continued arithmetic work and numerical computation throughout the secondary school period is recognized. . . .
>
> The applications of arithmetic to business should be continued late enough in the course to bring to their study the pupil's greatest maturity, experience, and mathematical knowledge, and to insure real significance of this study in the business and industrial life which many of the pupils will enter at the close of the eighth or ninth school year. . . . .

*Mr. Hardin:* What arithmetic would the Committee emphasize?

*Dr. Henderson:* The arithmetic to be stressed includes facility and accuracy in the fundamental operations; commonly used tables of weight and measure; facility in handling simple fractions; short cuts in multiplying and dividing; percents; line, bar and circle graphs; the uses of arithmetic in the home, community, bank, and in investments; and such elementary statistical concepts as tables and frequency distributions.

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[1]Copies of the report may be secured from the Dartmouth Press, Hanover, N. H. To defray wrapping and postage there is a nominal charge of twenty cents per copy.

Intuitive geometry is emphasized by the Committee. The older courses, as you know, gave little attention to measurement and space relations. The method of learning about the properties of space is that of experiment or intuition. Intuitive geometry should include, according to the Committee, direct measurement of distances and angles, the approximate character of measurement, areas and volumes of commonly used geometric figures, indirect measurement, the geometry of appreciation, simple geometric constructions with ruler and compasses, familiarity with the properties of such forms as the equilateral triangle, isosceles right triangle, the Pythagorean relation, and an introduction of the notion of similarity.

The Committee's statement of the results to be obtained from this instruction are quite to the point.

> The work in intuitive geometry should make the pupil familiar with the elementary ideas concerning geometric forms in the plane and in space with respect to shape, size, and position. . . . The work should, moreover, be carefully planned so as to bring out geometric relations and logical connections. Before the end of this intuitive work the pupil should have definitely begun to make inferences and to draw valid conclusions from the relations discovered. In other words, this informal work in geometry should be so organized as to make it a gradual approach to, and provide a foundation for, the subsequent work in demonstrative geometry.

What algebra would the committee emphasize? It recommends less algebra than was found in the older courses, with more emphasis upon meaning and use. The older courses offered little, if any, algebra before the ninth grade. Most of the course in that year consisted of manipulations of symbols that frequently had little meaning. Roughly one-third of the year was given to special products, factoring and fractions. The Committee recommends that the algebra to be taught in grades seven, eight and nine emphasize (1) the formula; (2) graphs and graphic representations in general; (3) signed number; (4) the equations—its use in solving problems; (5) the algebraic technique involved in the fundamental operations, in such factoring as (a) removing common factor from the terms of a polynomial; (b) the difference of two squares; (c) trinomials of the second degree that can be easily factored by trial; fractions; and such treatment of exponents and radicals as is required in the transformation of formulas.

Numerical trigonometry should be taught in the junior high school. Practically all European schools and hundreds of

American schools teach numerical trigonometry during the junior high school period. The National Committee recommends that our schools teach the meaning and use of the sine, cosine and tangent ratios. The committee's reasons for this recommendation are

> Its practical usefulness for many citizens; the insight it gives into the nature of mathematical methods, particularly those concerned with indirect measurement, and into the role that mathematics plays in the life of the world; the fact that it is not difficult and that it offers wide opportunity for concrete and significant application; and the interest it arouses in the pupils. It should be based upon the work in intuitive geometry, with which it has intimate contacts . . . . and should be confined to the simplest material needed for the numerical treatment of the problems indicated. . . .

*Miss Ryan:* Do we have *time* to teach numerical trigonometry? I was not exposed to trigonometry until my freshman year in college.

*Dr. Jones:* It isn't a question of time, Miss Ryan. It is a question of relative importance. You find time to spend three or four weeks on simultaneous linear equations, do you not? The Committee's arguments in favor of numerical trigonometry seem weightier to me than any argument I ever heard for simultaneous equations. The teachers in my summer courses who have had experience with this say that from three to six weeks are necessary for a laboratory development and treatment of numerical trigonometry.

*Dr. Henderson:* Are you going to teach any demonstrative geometry in your junior high school? The desire to provide pupils an opportunity to "explore" a wide range of mathematical subjects before completing the required courses has prompted some teachers to include a small unit of demonstrative geometry in the ninth grade. Concerning this practice, the Committee states:

> In regard to demonstrative geometry some teachers have objected to the introduction of such work below the tenth grade on the ground that with such immature pupils as are found in the ninth grade nothing worth while could be accomplished in the limited time available. These teachers may be right with regard to conditions prevailing or likely to prevail in the majority of schools in the immediate future. . . . .

*Mr. Hughes:* Yes, Dr. Henderson, I believe it is unwise to attempt demonstrative geometry in the ninth grade—even with bright pupils. My experience has led me to believe that it takes three or four months to accomplish anything really worth while.

I'm afraid we are going to spread out too much. Let's not attempt more than "is consistent with sound scholarship."

*Dr. Henderson:* Pardon Mr. Hughes, I had not completed the Committee's statement concerning demonstrative geometry. Continuing, the Committee says:

> On the other hand, it is proper to call attention to the fact that certain teachers have successfully introduced a limited amount of work in demonstrative geometry into the ninth grade . . . . . and that it would seem desirable that others should make the experiment when conditions are favorable. Much of the opposition is probably due to a failure to realize the extent to which the work in intuitive geometry, if properly organized, will prepare the way for the more formal treatment, and to a misconception of the purposes and extent of the work in demonstrative geometry that is proposed. In reaching a decision on this question teachers should keep in mind that it is one of their important duties and obligations, in the grades under consideration, to show their pupils the nature, content, and possibilities of later courses in their subject and to give to each pupil an opportunity to determine his aptitudes and preferences therefor. The omission in the earlier courses of all work of a demonstrative nature in geometry would disregard one educationally important aspect of mathematics.

*Miss Davidson:* What is your own opinion, Dr. Henderson, about teaching demonstrative geometry in the ninth grade?

*Dr. Henderson:* Personally I do not believe that demonstrative geometry belongs in the junior high school. If you recall, we accepted four major objectives or aims in mathematics, neither of which would call for demonstrative geometry. I think the Committee has pressed the *exposure* or *survey* or *exploratory* function of junior high school mathematics too far. We can't show pupils the "nature, content and possibilities" of the entire field of mathematics. Take the most wonderful of all subjects, the calculus. The Committee certainly would not attempt to give even a "peep" into it. Until I have more information concerning the value of "an introduction to demonstrative geometry" to the ninth grade pupil, I should prefer to omit it from our course. Let the experimental schools test it out more thoroughly and show us the results.

*Mr. Jewell:* How many of you agree with Dr. Henderson? All but one! For the time being, then, we shall omit the unit of demonstrative geometry. Won't you go on, Dr. Henderson, with the Committee's recommendations.

*Dr. Henderson:* You will be interested in what the Committee would *not* teach. Upon first consideration one might query whether the Committee had not recommended more material that can be successfully taught in these grades. Were it not for the

definite omissions that are mentioned, this query might rightfully be raised. Noteworthy among the omissions are: (1) highest common factor and lowest common multiple, except the simplest cases involved in the addition of simple fractions; (2) theorems of proportion relating to alternation, inversion, composition, and division; (3) literal equations, except such as appear in the solution of formulas, "including the derivation of formulas and of geometric relations, or to show how needless computation may be avoided;" (4) radicals, except as indicated on (page ); (5) square root of polynomials and cube root; (6) theory of exponents; (7) simultaneous linear equations in more than two unknowns; (8) the binomial theorem; (9) imaginary and complex numbers; and (10) radical equations except those that occur in dealing with elementary formulas.

*Mr. Hughes:* I think we should accept the Committee's recommendations concerning the topics to be omitted.

*Mr. Jewell:* Are we agreed on this? Not a single dissension!

*Dr. Henderson:* I have been expecting some one to ask about the sequence or order in which these various topics should be taught. The Committee made no attempt to formulate an ideal course of study, stating the proper sequence of topics by years or by semesters. It did, however, outline five "plans" in the hope that subsequent experimentation would help to determine the most satisfactory plan. In none of these plans is the entire work of one year limited to a single subject. Two, and usually three subjects are to be considered in each of the three years.

*Dr. Jones:* You have presented the Committee's recommendations for junior high schools. What mathematics does it recommend for schools organized on the eight-four plan?

*Dr. Henderson:* Schools organized on the eight-four plan should teach the same curriculum as junior high schools. The curriculum that is best for junior high schools is also best for the more conventional type of school. The committee argues this point vigorously, as follows:

> **The prevailing practice of devoting the seventh and eighth grades almost exclusively to the study of arithmetic is generally recognized as a wasteful marking of time. It is mainly in these years that American children fall behind their European brothers and sisters. No essentially new arithmetic principles are taught in these years, and the attempt to apply the previously learned principles to new situations in the more advanced business and economic aspects of arithmetic is doomed to failure on account of the fact that the situations in question are not and cannot be made real and significant to pupils of this age.**

I consider this one of the soundest of the Committee's recommendations.

*Mr. Jewell:* Yes, indeed. We have no doubt as to the validity of that point. It is to be hoped that schools in general will accept this thesis. What they most need is a reorganized curriculum, not just a change in the external aspects of administration.

Would it not be well to have a committee of our group draft a preliminary outline of the topics to be taught in each of the years? This committee will make generous use of the notes that we have taken this morning. I shall ask Miss Davidson, Mr. Hughes and Dr. Henderson to serve on the committee. We are adjourned.

# THE INTERPLAY OF MATHEMATICS AND ENGLISH[1]

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Until my very inspiring associations with the Mathematics teachers of the University, I should have said that the contribution of the department of Mathematics to the department of English lay in the correction of a child's inaccuracies of speech—the troubling verbs, and the no less troubling pronouns. But four years of delightful professional camaraderie with my friends, the mathematicians, have taught me that this correction of verbal errors, while it is a help for which all English teachers are grateful, is by no means the greatest service which the one department renders to the other. No child handicapped from birth by constant association with illiterate minds, ever learns to speak accurately just because he is required by teachers of all departments to stumble again and again over a recitation until he has straightened out his verbs and purified his pronouns. To a child handicapped by environment, accuracy of speech comes only as the result of an inward desire—a yearning, a craving, a passion, for accuracy. In the development of linguistic power as well as in the development of character, we are saved by desire and not by compulsion; we are saved from within and never from without. The correction of a specific inaccuracy, therefore, is a help, to be sure; but it is neither the only nor yet the greatest service rendered, as many of us in our blindness had once supposed it to be.

Much as some of us are offended by inaccuracies of inflection, we must admit that they do not present a serious social or racial menace; but inaccuracy of statement (whether accidental or intentional), false generalizations, ill-proved propositions, untenable assumptions, fallacious hypotheses, unfortunate rationalizations—all these are a menace to public good and to personal happiness. A child, therefore, must be trained in the proper methods of thinking, but he must not be made introspective.

[1]A paper presented at a joint session of the Kansas Association of Mathematics Teachers and the Kansas Section of the Mathematical Association of America, Topeka, February 7, 1925.

For this reason a high-school course in psychology seems altogether unwise. We must not, however, let children go through school without learning a few fundamental principles of thinking. Much of the responsibility falls—I think quite justly—upon the English Departments. But we do not need to carry the responsibility alone, since we have the mathematicians for neighbors.

Before we discuss this greater contribution of the Mathematics Department to the English Department, let me ask you to think with me of what task it is that the English teacher has set for herself. Mr. Dooley, in one of his charming conversations, says to his friend Hennessey, "I don't care what my son studies, so long as it is onplisent." And every teacher of English is convinced that Mr. Dooley referred to grammar. But grammar must be taught in order that a student may interpret the printed page with accuracy, and that he may test his own expression, either written or oral, and know whether it is right or wrong, and why. If the English teacher is wise, she will teach only the parts of grammar which function in the correction and interpretation of speech or writing; but if she is just, and honest, and courageous, she will teach every part thereof that does function in any such way. Grammar is science; to teach it well, one must be something of an analytical scientist, and know the methods of science teaching.

Then there is composition. We must teach it in the hope that therein a child may learn to clarify his own thinking as he masters the difficulties of expression. We hope to carry a child from the stage of the artisan—where his sole purpose is to utter without glaring blemish the thoughts that are in his mind—to the stage of the artist, where his desire is to say what he has in mind with the minimum of pain to himself and the maximum of delight to others. This is no longer science, but art; and the teacher herself must be something of a creative artist.

Then there is literature, which we teach very poorly indeed; for while we are quite sure what we ought to do, not one of us knows the technique. We know that for children literary study must be more than the mere accumulation of academic tradition, of names and dates and fine distinctions between ode and epode, much as these are to be desired.

Every teacher acknowledges that academic information is not the great end of high-school instruction in literature. What a man *knows* of the past or present in literature has little relation to his life. What he loves, what he understands, what powers of interpretation and application are his—these are the things that do make a difference. Every English teacher wishes that high-school instruction might make of all the children able critics of the trends in writing, and skilled judges of the product. Yet we all know that more important than training in artistry is training in attitudes; and that the end of literary education is not to make a child a brilliant or even a just critic, but rather to make him think in his heart what he ought to think.

Literature, therefore, must orient a child. It must teach him his proper and dignified place in the world of nature as well as in the social structure. When he is young, it must show him simple, obvious ethics, like truth and courage; when he is older, it must give him a sort of code of normal re-actions to experience with a supplement of re-actions that are better than normal. Let me illustrate: The desire to revenge an injury done us is a natural impulse. It belongs in our code. There is nothing wrong with the mind which in the first of a crisis desires it. But there is something wrong with the man who indulges that desire. There can be no further social evolution if we still ask an eye for an eye. The supplement to the code must tell a child that civilization is built upon self-control, and show him the methods of acquiring it. I speak of the code and the supplement separately for one reason. Children confuse the first natural impulse with what in the evolution of society we may call inherited repressions. This confusion is often disastrous. It often makes a child ashamed of an impulse when all he ought to be ashamed of is the failure to control that impulse; and he begins to fear that his mind is not working normally—although it is working perfectly normally both in his desire for revenge and in his desire to control that revenge. This fear of an ill-adjusted mind is by no means uncommon in high-school students. To allay these fears, literature must build up a code of natural impulses which a child should learn to recognize as normal and inevitable, and a supplement of social controls which a child must recognize as binding upon him if he is to do his part in creative evolution. Many of us find it easier to live in love and charity with our

neighbors than in peace with our own experiences. Literature must lay the intellectual foundations for this peace. Here we have more than science, more than art. We have—one is almost afraid to say it—we have here prophecy, not in the shamanistic sense, but in the sense that modern psychologists know it. Real prophecy is an interpretation to the people of what they want; it is guidance in the obtaining of that which they desire; it is the revelation of men to themselves. The teacher of literature, therefore, to be this prophet or interpreter, must be more than an analytical scientist, more than a creative artist; she must be a trained, keen, sympathetic, fearless critic, gifted with insight and common sense; and she must be, withal, something of an evolutionist and historian, something of a psychologist and philosopher, something of a humanitarian and a mystic—in the finest, sanest, most modern sense of mystic.

There has lived as yet no one who had within himself all the requisites for this trilogy of demands, no one who can be at once a great scientist, a great artist, a great prophet. Those of us who are at all skilled in self-analysis know where our weakness lies. If we are at all discerning, we learn to offset our weakness by building against the background that other departments have furnished.

For a wider and a non-sentimental understanding of nature, we must turn to the Departments of Natural Science, just as we must acknowledge our great debt to them for the particularly strong training they give in the organization and evaluation of thought. We are grateful to the Departments of Social Science for the training they give in temporal and spacial imagination, and for the furnishing of specific instances that make a vivid background for narrative reading and writing. Mathematics as it once was taught furnished almost no background of value to the English teacher; but Mathematics as it can be taught makes a three-fold contribution. First comes its contribution to the science of language.

When we must be scientists and teach grammar, we have the symbols of mathematics to help us explain verbal relationships. We all have difficulty in teaching children that "and" and "but" must connect like quantities; but the difficulty is minimized when they learn that "and" is the plus sign and "but" the minus sign of language, and that the rules for the mathematical

symbols hold good for the linguistic ones. The double negative is not a rarity. Of course we must correct it whenever it appears, but perhaps we ought not to expect its complete eradication until after children have learned in algebra to subtract minus quantities. That exercise furnishes a basis for explanation. Most of us who teach English feel that with high-school students correction without an explanation which is understandable, is vain.

The phrase punctuated as a sentence still occurs in the writing of children long after they have reached high school. Perhaps complete mastery here demands a maturity of judgment that we ought not to expect of all children in the ninth grade. Mastery is hastened by a study of the equation. I once heard a mathematics teacher say to the children, "$x - 2 - 4$" is not an equation, it is only an expression, because you have not said anything about it. You have not told us what it does. Add a predicate, $= 24$, and you know something about $x - 2 - 4$. It has meaning for you. You have a complete sentence, which in mathematics, we call an equation." This seems a very illuminating explanation of what constitutes a complete sentence. After a study of geometry a child is less likely to mistake a clause at the first of a sentence for a complete sentence. In geometry he has learned to recognize the clauses beginning with "when" or "if" as not expressing judgments in themselves, but as simply enumerating one set of conditions under which something else will be true. When a child has learned this, he is well on his way toward an understanding of sentence unity. Teachers of mathematics tell me that children find work in geometry much easier if they have learned to recognize a clause before they come to the department; and the English department finds that children, after an acquaintance with geometric procedure, punctuate their sentences with a much surer sense of unity and relationship. Here, it seems to me, educational economy would suggest that the English teachers train the child to recognize a clause before he goes into geometry, but that they be patient with him—though they correct his faulty punctuation—until his work in geometry has helped him to develop that most elusive thing—sentence sense.

Love of system the interconnection of judgments—has been called by philosophers the inmost essence of the intellectual impulse. The teacher of grammar, who is the scientist and the in-

tellectual among English teachers, feels that she has found an ally when she makes the philosophy of mathematics re-enforce the philosophy of language.

The second contribution of mathematics to English is made to the teacher of composition, who, though she should be a creative artist primarily, must face with her students two problems: the problem of mastering technique and the problem of testing the truth of one's thought. It is the veriest intellectual sentimentality to suppose for one moment that the average mind can ever learn any adequate control of an art without knowing something of the technique that lies behind it. It is the veriest emotional sentimentality to suppose that artistry is of any value without truth. To the technique of structure, mathematics make an interesting contribution. Every child learns in expository writing that a good paragraph consists of a topic sentence followed by description, explanation, or proof thereof. But most children, having written a topic sentence, are content with the flimsiest support for it. With the lovely certitude of the immature, they feel that this is sufficient—they have spoken. This fallacy is not confined to high-school students alone. Work with University intercollegiate debating teams is often one long struggle to force the debaters to develop and prove their points, to make them see that the unsupported statement of opinion or position is merely dogmatism and can be raised to the dignity of discussion only when the topic sentence is followed by adequate proof. Children in Junior high school ought to be taught something of proper paragraph development; but perhaps we ought to wait until after geometry is well under way before we upbraid severely the child who has not the requisite technique, for he will learn by the propositions in geometry the relationships of statement and proof.

Every good writer juggles with his sentences until the emphasis of expression falls on the emphatic parts of his thought. In this connection there is a most interesting exercise given in the new algebra—the change of the subject of a formula. If $S = 16T^2$, the formula may be made to read $T = \frac{\sqrt{S}}{4}$. By this change children see that they are able to state as an emphatic and isolated unit the symbol that represents the real question—the real prob-

lem. This change in subject is akin to our old exercises in grammar, where we changed the voice of the verb.

We used to struggle hard over this feat, and then tell children that when they had mastered its technique they must always avoid it, for the passive voice was dismissed as awkward and unseemly. But the passive voice is not awkward or unseemly. To be sure I am absurdly grotesque if, when I wish to say "She made the tea," I say instead, "The tea was made by her." Yet the fiber of a man's social qualities may be shown by the passive voice. After every storm this winter, when I have gone out with my shovel to clean my walks, I have found that some neighbor has been out before me. When this happens, I do not go back to my own fireside and merely say, "My walks have been attention is no longer on the domestic problem of cleaning walks; for I must solve a new problem: "Who was this someone?" My attention is no longer on the domestic problem of cleanng walks; it is on the social problem of ascertaining which neighbor to thank. In this case I am extremely ungrateful if the human element is not the subject of my thought. But every one's day is full of trivialities in which it is the veriest sentimentalism to insist that the human element must be the subject of thought. Have the ashes been emptied? When was the garbage collected? Was the milk left where it would not freeze? The unknown quantity in this case is not the human element. I know what that is. I am not concerned with it. I am concerned only with the question of work done or left undone. The passive voice is sufficient.

Again we may say, "He has always found courage in his friends," but how clumsy it would be to say, "Courage in his friends has always been found by him." Yet with true artistry we may say, "He has always been comforted by the comradeship of books," for the human element is stronger than the abstractions which ought to be subservient to it. We are not trying to solve the problem: "Of what use are books?" We are trying to give a formula for a man's reactions. This skilful use of the passive voice is one of the marks of the literary artist; but it comes only with a maturing power of evaluation. So I think that we who teach English ought to teach the passive voice as a grammatical form before children go far in mathematics, but I think we ought not to despair of the child who does not use it

skilfully until after he has seen in mathematics the reason for changing the voice of a formula. Several weeks ago in the English classroom the freshmen struggled with the scientific structure of the voices. A week or two later, they were taught in the mathematics classroom the reasons for changing the subject of the formula. Then they came back to the English class and we looked at the matter from the point-of-view of the artist. The departments must be learning the technique of joint instruction; never before have I found a group of students that mastered so easily this very difficult bit of technique.

Another exercise in algebra of interest to the teacher of English is the translation of an equation from the universal to the particular—the translation, for example, of $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$ into a concrete problem. This exercise is somewhat valuable because it gives an opportunity for the writing of a clear, definite sentence. But it is more valuable for quite another reason. The formula as stated is the essay of mathematics, for it states a general law capable of illustration by numberless specific instances. Here we are going from a general law to cases in which the general law holds good. That is just what a literary essay may do. The concrete problem is the fiction—the short-story—of mathematics. It gives a specific instance which can be interpreted and understood only by recourse to the general law. It is merely one illustration of the working of that law. That is true of the short-story in literature.

Teachers of mathematics tell me that children have great difficulty in devising the problems to illustrate the general principle. Every English teacher knows that children in the twelfth grade often write very creditable and rather mature essays; but a good short-story is almost never produced. In the last year-book of one of the special experimental high-schools, there appear poems of high grade, essays of unusual insight, but not a short-story that by any intellectual generosity could be called good. Since the Mathematics and English departments have almost parallel experiences with their short-story work, it would seem that there must be some explanation behind the difficulty. Dr. Coe—in answer to an entirely different problem—has perhaps given us the explanation of our own. "Metaphysics," he says, "through-

out its history has found the universal easier to handle than the particular, especially easier than the individual." This has a significance for English teaching. From it we know somewhat better than we did what to expect of children; we realize that the inability of the child to turn out a short-story comparable with his essay is not a reflection upon his diligence nor yet upon his instruction. Perhaps, too, we shall realize that literary honors are more justly bestowed than we had supposed. Perhaps the man who writes a masterpiece in fiction deserves greater compensation and greater devotion than the man who has cast his thoughts in the easier mold. It is always salutary for us all, young or old, to realize that the treasures and rewards of life are justly bestowed.

The third contribution of Mathematics to English comes when the teacher must be a prophet and interpret children to themselves. Here comes first, perhaps, the responsibility for equipping the child with an adequate acquaintance with the meaning of words. There are dozens of purely mathematical terms which have in them what the journalists call "the human element." Without mathematics, how can we understand what it means to "go off on a tangent," to "have all interests inscribed in the tiny circle of our home"? When the mathematics teacher is first presenting these geometric relationships, she can explain, much better than can any English teacher, the symbolism which literature borrows.

There is, also, the necessity of teaching children to read for the central thought—the theme—the main point of the whole work. And when the child has found this theme, he must be able to state it in general terms so that he can lay it aside in his mind for future illustrative purposes. This is what, of course, you who are mathematicians call generalizing. You can teach the spirit and practice, the necessity and method, of this art much better than can we who teach English. During one of my frequent visits to the geometry class-rooms, I heard a boy stumbling through the reasons why two triangles were similar. The reason for their similarity he stated in specific rather than in general terms. His teacher explained to him that his conclusion in this case was correct: the two triangles were similar. But information, she explained, was rarely of value unless it could be stated in general terms so that it could be used again. The incident.

so far as I was concerned, dropped there. I was not yet ready to talk of the necessity for literary generalizations. A few weeks later, I said to my group, "What is the central thought of *Silas Marner*?"

"It is a story," answered one of my lads, "of a man who ran off because he was falsely accused of a theft, but he didn't clear himself that way."

"That will never do," one of the girls insisted. "You can't re-use your information unless you put it in general terms. You'll have to say something like this: *Silas Marner* proves that when a man is in trouble, he cannot solve his problem by dodging the issue." Here is at least one instance of the transfer of training.

Just as mathematics must assume the chief responsibility for training children in the necessity and technique of generalization, so it may train children in the spirit and practice of symbolism. Voltaire once told us that poetry can say more, and say it in fewer words, than can prose. And Dr. David Eugene Smith tells us that mathematics and poetry have that characteristic in common. $(a + b)^2 = a^2 + 2ab + b^2$ is more easily remembered, more easily used again, than is the same truth written in words.

"In the Cross of Christ I glory,
Towering o'er the wrecks of time."

is more easily understood than the same truth worded in prose. Here we have three symbols which must be resolved into reality: the Cross—the towering—the wrecks of time. Solving these into the most reverent of realism we get a cumbersome thing like this: "I rejoice very much in the life of Christ—His teachings, His examples, His social outlook, His sacrifice, His resurrection—which have survived the vicissitudes of twenty centuries, which have been, and still are, the most powerful influence in history." Our minds may give assent to this argument, but it is assent uncompanioned by either joy or gratitude. We cannot do without poetry—and by poetry I mean not verse alone but symbolism whether rhythmic or not. Our accuracy of thinking both secular and religious depends upon our skill in distinguishing between the realistic and the symbolic, and upon our respect for both. The mathematician who writes upon the board the words of Vol-

taire and the comment of Dr. Smith, has done much to help the child who respects what he calls the hard, cold facts of mathematics, respect what I should like to call the warm and living truth of poetry. When he respects poetry, he will learn somewhat to understand it.

Maria Mitchell once said that a formula was the hymn of the universe. She did not say it was a religious song. She said a hymn—for a hymn is the cry direct to God either for confession, for protection, or thanksgiving.

"Dear Lord and Father of Mankind,
Forgive our feverish ways."

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"Jesus, Lover of my soul,
Let me to Thy bosom fly,
While the nearer waters roll,
While the tempest still is nigh."

* * * * *

"We praise Thee, O God,
We acknowledge Thee to be the Lord."

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The religious song is a different matter. It is usually nothing but social advice set in verse form—"Throw out the life line"—"Rescue the Perishing"—or the worse things like "Get aboard, get aboard, get aboard the Gospel Ship," which have grown out of the absurd doctrine that to democratize art, or religion, or education, one must reduce all fine things to a minimum of fineness. This sort of social advice or exhortation is always transitory and fleeting in its appeal because it is usually trivial, useless, and insincere. It expresses no emotion, no need of the singer. It has in it something of the aspect of the superiority complex. The hymn is quite different. It expresses only the feeling or the need of the one who sings. And the need which it expresses is universal—the need of protection, forgiveness, and thanksgiving. The formula is indeed a hymn, for it ex-

presses universal truth, discovered, it always seems to me, with a cry of triumph and thanksgiving that the long warfare with that bit of ignorance is accomplished. When the time comes that the mathematics teacher puts on her board the words of Maria Mitchell, the English Department begins training the children for some sort of discriminating evaluation of religious poetry.

To the teacher who is trying to be something of a prophet for the children, there often come some of the most difficult questions about life and God, about suffering and immortality. When we are trying to give children an idea of the justice of God, nothing helps us more than does mathematics with its insistence upon the permanency and the universality of law. This procedure is sound, a philosopher tells me, even judged by our best doctrines of the Theory of Knowledge. Perhaps no single study helps more here than does Dr. Smith's presidential address on "The Religion of the Mathematician"; for to meet the questions that come, we need the wisdom of all philosophies and the argument of all sciences.

This study has been an attempt to outline but a few of the points at which Mathematics and English meet. I am quite as ready as you to acknowledge that herein there is nothing scientific. We have had no conscious investigation. We have had no set program for observation, no control groups, no adequate facilities for experimentation. We have been merely watching the ground with interest. But I would hope that out of this there might come something worth-while. I would hope that somewhere a Mathematics teacher and an English teacher who have faith in their own work and sympathy for the work of each other, might be given an opportunity to make a real study. Then when we speak of the interplay of Mathematics and English, we shall have something finer to record than the merely casual observations of one teacher of English.

Said the wild waves "Let us spray"

# THE VALUE OF THE HISTORY OF GREEK MATHEMATICS TO SECONDARY SCHOOL TEACHERS

By WALTER CROSBY EELLS
Whitman College, Walla Walla, Wash.

The course in history of mathematics is given this year at Whitman College two hours a week for sixteen weeks to a class of five juniors and seniors, all of whom are planning on teaching high school mathematics. At the end of six weeks, upon the completion of the study of Egyptian and Greek mathematics, the following question was asked as part of a test covering the subject:

**What value have you derived, as a prospective teacher of high school mathematics, from the study of the history of mathematics up to the present stage, i.e. to the end of the Greek period?**

Only a few minutes were taken for the question and needless to say the answers were not intended for publication. They lack the perspective that would be obtained from the completed course. The period covered is that of the beginnings of the science. Yet they are so interesting and significant that they may be of interest to present teachers of mathematics. Possibly they may suggest to some teachers who have not had the benefit of such a course the desirability of studying it by themselves in one of the excellent books now available.

In addition to the well known histories of W. W. R. Ball and Florian Cajori, they are fortunate now in having available the exceedingly interesting and profusely illustrated two volume history of elementary mathematics by David Eugene Smith, of Columbia University, which has been used by the class this year as a text. For the particularly significant Greek period there is also the scholarly and authoritive two volume "History of Greek Mathematics" by the Cambridge scholar, Sir Thomas Heath.

The replies to the questions, exactly as written, as as follows:

**In the first place, I think that I could now teach mathematics with much more vigor and enthusiasm for it now seems more human to me. I can now see it as a logical development and hence look upon it as a natural thing. Knowing interesting incidents in the lives of the original makers of certain theorems will help make that theorem more interesting, and knowing how certain crude methods were tried and then failed and then how the result finally came about, would make a theorem more interesting and easily understood by a class. I now have a general survey of the whole beginning of mathematics which helps in the understanding and appreciation of any sub-**

ject. There are now many questions which were formerly unanswered in my mind which now seem to be answered. Such an outlook as this seems to be neccessary in the teaching of any subject.

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The theorems and problems that we have studied mean more to me than they have before and I understand them better. It helps to show that the proofs in our geometry books are not the only ways of proving the different propositions. By knowing the historical facts connected with the different prpositions you can explain them better.

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Values derived from this study so far:

1. I have learned that knowledge did not develop as it is taught to-day. Our spherical trigonometry does not precede our plane. From this it might be questioned whether our order of study is the best; whether a correlation of the different branches might not, at least, be as good.
2. The old proofs of the original theorems are interesting to us and might prove enlightening to high school students in order to give a broader point of view and a larger field to work from.
3. The conditions under which these first theorems arose would be interesting to high school students.
4. The mention of the old mathematicians and how they were connected with their respective theorems would be good in teaching.
5. This study also shows us how superior our system of proving geometrical theorems is, and how good our algebraic symbolism is. The students should be made to appreciate this fact and to be modern and up to date with all their work, making it complete but compact.

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In the study of Greek mathematics I myself have received many new ideas about the origin of mathematics and I think that in teaching high school mathematics some of these facts might be cited in the presentation of various problems to make them more interesting. I think that historical facts of this nature given in connection with different problems makes the student remember them better.

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The study of the history of mathematics is to me and should be to any mathematics teacher or student, I think, a wonderful inspiration. Facts of mathematics are no longer just facts, but are living, growing thing as compelling in their fascination as any other life process. All high school students are interested in progression of things, movement is an essential of their lives, and if they can be made to get the spirit of the development of mathmatics, it undoubtedly would be a great stimulus to their interest. The hueristic spirit of a teacher and his pupils can best be kept alive by continual study of the history of his subject.

Another question asked was: "Who is the most interesting Greek mathematician you have studied, and why?" It may be of interest to know that four students voted for Euclid and one for Archimedes. One characterization of each is quoted.

I consider Euclid the most interesting Greek mathematician because of his ability to compile all the then known matter on geometry and to put it into such a form that it has been and still is used as a textbook on geometry. Surely he was most interested of them all in progress, as his attempt was to place the knowledge of mathematics within reach of all future generations.

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The most interesting Greek mathematician to me has been Archimedes. I suppose it is because he did so many original and wonderful things that I find him so interesting. His work is so varied and there is so much in it that could be used to interest classes in high school mathematics. I have a feeling that I should like to know Archimedes for he seems so human and his work seems to describe what he must have been like.

# THE ECLIPSE

By FRANK J. McMACKIN
Dickinson High School, Jersey City, New Jersey

On the morning of Saturday, January 24th, 1925, there occurred a spectacle in the Heavens which few men are fortunate enough to experience even once in a lifetime. I was one of the elect on this occasion. The world awoke that morning a stage perfectly set for the wonderful sight nature had arranged. The atmosphere was absolutely clear, no trace of a cloud was visible anywhere and the sun seemed to be unusually bright—flushed with excitement over the performance it and the moon were to enact that day.

The audience, too—some 20,000,000 persons—had prepared itself. Business men, realizing the possible effect it might have on their employees, closed their establishments that their employees might take advantage of this event. Thousands of dollars and the labor of some of the country's most learned men were spent in the anticipation and study of this phenomenon. The newspapers for weeks before carried notices of its coming, instructions as to how it might best be observed and stories of its past performances and future occurrences. Many persons went miles to be in the area of totality that they might better enjoy—what? Most of them could only guess, but they expected something very great—wonderful—unique—and they experienced a sight greater, more wonderful, more unique than their most extravagant imaginations could formulate.

Many of these 20,000,000 had planned to arise that memorable morning at a certain hour that they might be on time. Most of them had fortified themselves with bits of smoked glass or old films that they might be ready at the appointed hour, for this spectacle was to occur not merely on January 24th, but at a certain hour on that day, yea, more, at a certain number of minutes and seconds after 9 o'clock on that day.

Can you think of another age when men arose from their lowly beds each morning, tended their animals and fields throughout the day and retired again in the evening? Think how their same tasks recurred day after day, only changed by the changing seasons. Then try to see them arise on some beautiful, bright, sunny morning, as was their usual custom—see them on this particular morning, tend their cattle and feed their horses—

and see them out in the fields busily engaged with their usual tasks but a short time when the light of day begins to fade, see the curious expressions on their faces when they realized it was growing dark before high noon with no clouds to hide the sun. Think, if you can, of the fears, the superstitions, the imaginings, that might be set up in their simple, albeit ignorant, minds as they see the birds take to their nests, the stars appear in the daytime sky, the face of the sun completely hidden, and, worst of all, to see the very trees and fields around them begin to quiver and dance. What could these men have thought? Why the great difference between these people of a few centuries ago and the people of today in a matter of this sort? The answer might be given that it is the usual difference between an ignorant people and an educated one. But there were educated men in those days. Still, there is a tremendous difference. It is not the difference between ignorance and enlightenment that is evident here, it is the difference between being non-mathematical and being mathematical. Educated men might, if they had kept careful records over many thousands of years have been able to say that the eclipse would occur on or about January 24th, 1925, but we were told the moon would completely hide the sun at 22 seconds past 11 minutes past 9 o'clock on January 24th, 1925. This is not the difference between ignorance and learning—not the difference between non-recording and recording—however accurate —*It is the difference between being non-mathematical and being mathematical.*

Astronomy is a science for no other reason than that it has become mathematical, or, rather, that it has become mathematicized. Here is a striking example of the worth of mathematics. Through but one minor application of this subject, to but one branch of thought, 20,000,000 men and women were made to attain as nearly perfect happiness as men can hope to attain. But this is only one of the millions of uses to which mathematics is put to-day. In fact, every phase of human thought, every field of human endeavor, every body of human doctrine, is striving to become mathematical, is only waiting to become mathematicized. Men in whatever field they might find themselves, who are able to carry their particular subject even a little way along the road to becoming mathematical, will be the big men in their field.

## NEW BOOKS

**Boys' Own Arithmetic** by Raymond Weeks. Published by E. P. Dutton & Co., New York, 1924. Pp. XVIII—188. Illustrated by Usabal.

The favorite sport of teachers is to establish plausible alibis for their many failures. The writer of this review, a hardboiled mathematician with casualties of a quarter century to his credit, first tackled University teaching, where the inefficiency of preparatory schools formed the theme of daily lamentations. Later on, after many years of experience in secondary school work, he traced the tap root of the difficulties down to the primary grades. Seriously speaking, if a normal child be efficiently taught in the rudiments of Arithmetic, he has a tremendous advantage when wrestling with the higher phases of Mathematics.

The problem is, however, more complex. The reviewer has in mind a little ten-year-old girl who was recently most unhappy in school. Of extraordinary intelligence, with an I. Q. that placed her in the ranks of thirteen-year-old girls, she failed on account of that miserable subject, Arithmetic, and spent her days in worry and tears. The reviewer consoled her with the promise of a gift that he guaranteed would make her laugh every time she tackled a problem. He gave her a copy of Raymond Weeks' "Boys' Own Arithmetic."

Perhaps a change of teachers may have accelerated her sudden marked improvement; but he will always believe that the book itself must receive appreciable credit. It changed her point of view. She no longer feared the bugaboo that had been made the object of delicious irony by the famous professor of Romance Languages at Columbia, the spirit of which she grasped with uncanny intelligence. Though conscientious as ever before, she had obtained a viewpoint of relative values, and failure in that subject receded as she regarded it with calm indifference.

It would pay us all, old and young, teacher and pupil, to read the "Boys' Own Arithmetic," and laugh uproariously at its jolly caricatures of old style textbooks. We would develop greater sympathy for both teacher and victims. We would more clearly realize the difficulties of an abstract, exact subject which must

later on be applied to life, filled with compromises and nebulous data. Dodgson, the Mathematician, became the famous Lewis Carroll when he sought relief by occasional flights into the realms of nonsense; and none were more astonished than he at the world-wide reception given his imaginative "Alice in Wonderland."

Professor Weeks has taken an even bolder course in straying long enough from the attractive fields of Romance Languages for a light-hearted excursion into the gloomy territory of numbers. He has conquered the Mathematics teacher without a painful struggle. How can one set up a defense when one must laugh heartily at his witty sallies and secretly agree with him that the teaching of arithmetic is sadly in need of reform?

So read the book by all means. It gives a humorous, kindly slant to life. It furnishes food for thought to philosopher and prosaic ones alike. A daily dozen of hearty laughs are worth more than a whole bag of dry-as-dust number problems, and what between the one hundred and eleven problems and the more than fifty inimitable illustrations by Usabal, the book is laughter from cover to cover. Here are two or three of the briefer problems:

"On the Manakin Road lives a Dog who barks for 40 minutes whenever someone goes by at night. What is the smallest number of passers-by that will keep him barking all night? Allow 10½ hours."

"An Opossum intends to eat 3 Persimmons every other half-hour during a night of 11½ hours. Being weak in Arithmetic, he makes an error, and eats 4 Persimmons every other 20 minutes of the time indicated. If each Persimmon gives him 1½ pleasure-pounds, how much does he gain or lose from his deficiency in Arithmetic during the month of October?"

"Mrs. Superbia McManus, age unknown, having little to do, joined the Browning Club of MacIvorsville. On her way to attend the second fortnightly meeting for December, she slipped and fell in front of the well-known Tonsorial Parlors of Lonnie Frazell, and had to return home for repairs. She was 43 minutes late in arriving at the meeting. During every 1⅓ minutes of the time she was late, one-sixteenth of her character disappeared. How much remained?

G. W. MARQUE MAIER,
Instructor of Mathematics,
The Peddie School, Hightstown, N. J.

**Corrective Arithmetic.** By W. J. Osburn. Houghton-Mifflin Co., Boston, 1924. Pp. 178.

A very comprehensive piece of educational research has been made recently by Dr. W. J. Osburn, the Director of Educational Measurements in the Wisconsin State Department of Public Instruction. Although the study itself covered a broad range of subjects the material which best lent itself to organization and publication was in the field of arithmetic. Dr. Osburn's procedure included an analysis of errors found in problems, and their causes, and culminated finally in an adjustment of remedial measures to these errors.

One of the biggest assets of the book is the fact that it is most specific. The author does not indulge in generalizations; he has definite, sometimes almost startling information, as to our present lack of efficiency and he presents this along with a suggested solution of the difficulty. It is Dr. Osburn's opinion that most of our troubles in arithmetic are a result of our inherited faith in the efficacy of transfer, sadly misplaced as it is. After analyzing the processes as wholes to determine the relative value of subject matter with a view to the needs of the pupils, and describing the types of practice material necessary, the author outlines the work grade by grade, concluding with a discussion of method which includes some helpful games and devices.

The book will be useful to all those interested in any phase of the teaching of arithmetic. For those in the field of measurements it is a contribution of considerable importance and for those looking at mathematics from the classroom viewpoint it presents a foundation on which they may well stand.

Josephine Haley.

The Lincoln School.

**La "Thiende" de Simon Stevin.** Fac-similé de l'édition originale de 1585, avec une introduction par H. Bosmans, S. J. Antwerp, 1924, pp. vi. + 37. Price 15 Belgian francs.

To those who recall Miss Sanford's translation and study in The Mathematics Teacher of October, 1921, there will be special interest in knowing of this publication, issued by the Société des Bibliophiles Anversois, being No. 38 of the series of this society. It seems that shortly before the burning of the library of the University of Louvain, Père Bosmans had borrowed the

book and had taken it with him to Brussels, thus saving it from destruction. This first edition appeared in 1585 at Leyden, in the Flemish language, and now is very rare, only a few copies being known. It was immediately translated into French, appearing as a part of *L'Arithmetique de Simon Stevin de Brvges* which was published at Leyden in the same year, and by the local branch of the same press—that of Christophe Plantin (Christoffel Plantijn).

Naturally our American readers will prefer to consult Miss Sanford's translation or the French edition, but it will not be without interest to possess such a valuable historical work in fac-simile, forming the first printed treatise on decimal fractions, particularly since the Société des Bibliophiles has issued it at a merely nominal price. It may be purchased of the Society at Marché du Vendredi, 22, Antwerp.

DAVID EUGENE SMITH.

**Solving Geometric Originals** by Frank Charles Touton, Bureau of Publications, Teachers College, Columbia University, New York City, 1924; pp. 114.

This study, made in 1919 and published in 1924, is an attempt to deduce from the solutions of exercises in geometry those principles which should guide the teacher when directing "the thinking of pupils in their first attempts at the solution of new and involved geometric situations." The basis of this study was a critical analysis of about 2,800 papers in Plane Geometry of the New York Regents Examination, June, 1918.

The introduction consists of a list of the facts and principles involved in the proof of each of the eight exercises studied, but treated as separate ideas or concepts with no hint as to their interrelations.

In order to examine the relative difficulties of the exercises, the author assumes "that other factors so counteract one another that their combined influence would not greatly disturb the results obtained from the use of the mean scores and the percentages of the pupils selecting the exercise." In a recent experiment the reviewer found that other factors, including the interrelations mentioned above, are the essential parts of the proof wherein lie the various degrees of difficulty. Professor Touton also states that teachers by virtue of their own analysis of exer-

cises should weight these according to their relative difficulty. However, teachers do not seem to agree on even the relative order of difficulty of exercises involving demonstration of a given statement; this was shown clearly in a study reported in the February issue of the MATHEMATICS TEACHER, under the title, *Difficulties in Exercises in Geometry.*

The sex differences in mean and median scores were found to be very slight, favoring the attainment of boys. In fact, the differences are so small that we wonder if their standard errors would not reveal them to be actually, or nearly, chance differences. The author himself cautions readers in interpreting this data in that the papers examined included practically none of less than passing grade. Furthermore, when this data concerning the variability of the sexes conflicted with the findings of other investigators, the author states the comparative data with considerable care, as well as his interpretation of the causes underlying these differences.

Chapter IV describes the "Steps in the Thought Processes Involved in Solving Originals." The importance of the knowledge of basic facts and principles in relation to their probable uses in the proofs of exercises, of the potency of elements, of testing suggestions as they arise to select those pertinent for this exercise, and the final step of logical arrangement of the steps in the proof, are very carefully discussed. However, the "search for direct outcomes" of the given and required elements places the emphasis upon inductive, rather than the deductive reasoning of geometry. This inductive approach and the tendency to regard the given rather than the required elements as the guiding factors in the solution of an exercise result in a student technique which leads into various attempts each with apparently equal probability of success. On the contrary, a student technique based upon the deductive approach with the required relation as the guiding factor lessens the process of trial and error, because it is the means of indicating which one of the possible attempts has the greater probability of success. This process facilitates the formation of habits of reasoning more directly from the known to the unknown relations.

Chapter V contains many suggestions of practical value for the classroom teacher. But, in the light of more recent investigations, the suggestions concerning the type of classroom par-

ticipation of superior students as well as the extent of classroom drill of propositions are not advisable.

Thus, *Solving Geometric Originals* is a very careful and detailed study of certain aspects of the results of a year of study of Plane Geometry by a selected group of students. It is not an investigation of the actual learning process of a more typical group or of individual students, as "solving" might imply.

WINONA M. PERRY.

April 8, 1925.

**History of Mathematics** by David Eugene Smith, Ginn and Company, Boston, 1923 and 1925, 2 vols. Vol. I, pp. xxii + 596. Price $4.00. Vool. II, pp. xii + 725. Price $4.40.

A work such as Dr. Smith's *History of Mathematics* should be discussed from two points of view: that of the scholar who can appreciate the breadth of vision and the carefulness of detail of these volumes, and that of the class room teacher who is insensible to many of the finer points of the workmanship, but who sees here a valuable aid in the task before him. Most of us will judge it from the latter standpoint.

The value of the history of mathematics to the teacher rests on the postulate that familiarity with the steps in the development of any subject increases our enjoyment and appreciation of that subject. We all feel a bit superior to Peter Bell who saw nothing but a common yellow flower in the primrose, but there are many of us who are in precisely the same position with regard to the ordinary things about us. This is particularly true of our attitude toward mathematics. We tend to assume that number systems, symbols, equations, etc., have always been as they now are. When we study their evolution to their present form, however, we find that they gain in interest. Furthermore, this study gives the teacher a better understanding of the difficulties of a child in learning to use these things, and it increases the teacher's idea of the significance of the topics he teaches.

We have given much attention to the *practical* aims of the teaching of mathematics, and we have investigated the *disciplinary* ones, but while the *cultural* aims are the subject of frequent discussion, we have done little about them. Of course, practical objectives are relatively concrete. Disciplinary factors can be measured. Cultural values, on the other hand, seem to be neither

tangible nor measurable, yet we are quite sure that they cannot be attained inadvertently. The history of mathematics is one of the surest roads to this end. A child's appreciation of the meaning of the subject is greatly increased when he sees it as a body of thought that has had its periods of rapid growth and its times of apparent stagnation. His interest is caught when he learns that men of whom he has heard in other connections had their part in its development. He is often anxious to know why his geometry shows him how to solve algebraic equations by construction with compass and ruler when he finds it more convenient to solve the equation by his accustomed algebraic means. He is sometimes inclined to wonder how any one happened to think of a logarithm but he is generally more apt to think that people were very stupid to have taken so long to invent things that are matters of course to him. The discussion of the history of the topics that we study in elementary mathematics, therefore, is not a mere time-killer. Its function is to show the subject as a thing closely associated with the development of human thought and that by virtue of its very human-ness it shows the same inertia, and the same love of things we are used to as characterize man's attitudes in general. The surest way to show a class that their views on the immediate adoption of the metric system are impossible of fulfillment is to tell them how people hesitated to adopt decimal fractions, and how even our own numerals were once so novel that shop-keepers in Florence were forbidden to use them in marking their goods, but were told to use "clear letters," i. e. Roman numbers, instead.

Unfortunately, this cannot be given to one's pupils in set doses. It is most effective when they beg for it. It is astonishing, however, to see the number of "whos," "whens" and especially "hows" that arise spontaneously and to see how the questions increase in frequency and thoughtfulness as they meet with a reasonable response. The history of mathematics, then, is like many other assets, a tool that cannot be used for all purposes, but that suits certain particular occasions as nothing else can.

Of the general histories of mathematics in English, none fits the requirements of teacher and student so adequately as does this new publication by Dr. Smith. Three factors obviously contribute to this: greater length and therefore greater detail than we find in either Ball or Cajorie, a dual method of presen-

tation, and the great number of illustrations. None of these, however, suggests the most important thing about the two volumes. They could never have been written except by one who knows and loves books, one who has travelled extensively and observantly, one whose interests are not narrow. Even the casual reader will be impressed by the scope of this work, and the earnest inquirer will appreciate its uniqueness.

These volumes are at once more scholarly and more useful than the other general histories of mathematics. It is unfortunate that good printing and many illustrations are costly, for the price of this work will, one fears, limit its purchase by the people who need it most.

In Volume I the *General Survey*, we have a discussion of the topic in its broader aspects. The book is devoted to the development of mathematics in the different parts of the world at different periods. In this, the contributions of specific persons are given with such details of biography as are needed to make these men real characters not lay-figures. To those of us whose historical interest is largely bounded by Europe and America, the emphasis on the mathematics of China and Japan may seem excessive. One must own, however, that this prejudice is due to our small-mindedness rather than to the merits of the case, and that the inclusion of these chapters enables us to contrast two different developments of the subject. The chronological table of this volume and the maps that are given are a great help to the student, and the introductory chapter on the pronunciation of proper names is a valuable addition.

Volume II, the *Topical Survey*, provides an answer to such questions as "When did people begin writing equations as we do now?" "Where did our radical sign come from?" "How did people write trig tables before they knew about decimals? "Did anyone ever seriously use similar triangles to measure inaccessible distances?" etc. This volume is devoted to a detailed treatment of topics in Arithmetic, Theory of Numbers, Algebra, Geometry, Trigonometry, Analytics, and Calculus. Such an arrangement makes it a tremendously useful reference book.

The two volumes are supplementary to one another. The first gives the big things in the history of mathematics as a whole so that we have a picture of the contributions of different periods

and a notion of the status of the subject in each of them. The second volume follows the history of the details.

Both have indexes that work.

Each has many illustrations: portraits, facsimiles of letters and other biographical material, pictures of mathematical instruments, and, most useful of all, pages from books and manuscripts showing how various things looked when they first appeared in print, or how the work of earlier times was written. As very little of this has been reproduced previously, the author has given us a unique collection whose usefulness will be guessed by those who have seen the originals of these cuts.

It should be noted that this history furnishes no denatured—highly-popularized account for one who seeks mere entertainment, but it gives a discussion that is intelligible to the lay-man who believes that the history of mathematics is worth consideration as apart of the history of civilization. One should guard against a feeling of discouragement at the quantity of material that is presented here. There is too much of it for any one to hope to grasp it all. If one considers it as a reference book, he will find it more and more useful; if he expects to master it at one reading, he is fore-ordained to failure. The work belongs in Bacon's third category: "Some books are to be read only in parts; others to be read but not curiously; and some few to be read wholly, and with diligence and attention."

VERA SANFORD.

## NEWS NOTES

*To the Members of the Central Association of Science and Mathematics Teachers and to other teachers of Mathematics in the Middle Western States:*

Our next Thanksgiving meeting will be held at the University of Chicago on Friday and Saturday, November the 27th and 28th. We are very anxious to build a strong program and we believe that this matter should not be delayed until next fall. The members of this association can be of great help to the Steering Committee by sending us the names and addresses of teachers who probably have material that could be used in this program. We would like to put the emphasis upon investigational matters. If you know of any teacher who can make a contribution to this program, be sure to send his name to us.

RALEIGH SCHORLING,
President Mathematics Section,
Ann Arbor, Michigan.

*To the Members of the National Council of Teachers of Mathematics:*

The meeting at Cincinnati was a most successful one. Those of you who have read the last two numbers of the Mathematics Teacher are probably convinced that the various papers read and addresses delivered were unusual in merit. If we are to have a program equally good next year, we must begin now to build it by co-operative effort. No one person or even a committee can build the best program. If there is any one in your section of the country who is working on a problem and who can organize effective material and present it at the next meeting of the National Council in February, 1926, we should like to have the name of that person as soon as possible.

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RALEIGH SCHORLING,
Ann Arbor, Michigan, President National Council
of Teachers of Mathematics.

The fourth annual meeting of the Inland Empire Council of Teachers of Mathematics was held at the Lewis and Clark High School, Spokane, Washington, on Wednesday and Thursday afternoons, April 8th and 9th. The meeting was called to order by President N. J. Lennes of the University of Montana. The members for the year 1924 answered to roll call.

The program for Wednesday was a very interesting report given by Stephanie Berthot, Missoula County High School, Missoula, Montana. In this paper Miss Berthot gave an historical resume of the recent efforts to improve the teaching of algebra. This proved very enlightening and a very good preparation for the report that was given the next afternoon by Prof. W. C. Eells, Whitman College, Walla Walla.

Last year Prof. Eells was made chairman of a committee to act as a clearing house for the administering of the Hotz Standardized Algebra tests in the High Schools of the Northwest. These tests were given in approximately 75 High Schools of the Northwest and the results sent in to Prof. Eells. The committee then worked out a very extensive report comparing the various schools, the results in respect to the text book used, comparing the large, medium and small schools; comparing the achievement of boys and girls.

After this report the Council voted to continue the committee and to give the tests another year. If the committee considers the matter feasible standardized tests will be given in Geometry also.

CHRISTINA CLAUSSEN,
Secretary of the Enland Empire Council of Teachers of Mathematics, Lewis and Clark High School, Spokane, Wash.

April 18, 1925.

The Mathematics section of the Ohio Educational Conference met in Columbus, April 3, 1925. Mr. Robert L. Fleming of Youngstown spoke on *Mastering the Fundamentals of Plane Geometry;* Professor K. D. Swartzel of the University of Pittsburg discussed *Some Collateral Values in the Study of Mathematics,* and C. F. Geeting of Dayton read a paper on *Increasing the Interest in the Study of Mathematics.* J. C. Bold of the Steel High School, Dayton, presided.

The *Chicago Schools Journal* for March has more than the usual interest for teachers of mathematics in junior high schools.

Professor Hinkle in *Arithmetic in the Junior High School* emphasizes the importance of the subject and its value in training pupils. He corrects a fallacious notion regarding junior high schools when he says that the junior high school is not to shorten but to enrich school life. He also says that our greatest concern should not be the bringing of topics to the junior high school which properly belong to the senior high school, but to teach more effectively such topics of arithmetic, algebra and practical geometry as are easily within the comprehension of the pupils.

Maud Harrington of the Park Manor School in *A Pageant of the Numbers* gives interesting suggestions as to securing motive and interest to pupils in the study of mathematics. *From Decimal Point to Percentage Symbol* by Sayrs A. Garlick of Fenger High School gives valuable suggestions regarding the teaching of a topic which frequently offers unusual difficulty to the pupil.

A. D.

Under the heading *Mathematics and Eternity* appeared the opening article of the January, 1925, number of the *Mathematical Gazette*. The article was written by one of the most noted women mathematicians of our generation, Miss Hilda P. Hudson, London, England, who was a member of the Council of the London Mathematical Society from 1917 to 1923, and published in 1916, a useful book entitled "Ruler and Compasses."

As this article emphasizes aspects of mathematics which seldom receive attention in journals devoted to the interests of "Teachers and Students of Elementary Mathematics," we quote therefrom as follows:

But however we think of Heaven, it is hard to imagine astronomy and botany surviving as they are, and having such interest or importance there; for they deal with the behavior of matter in terrestrial conditions, and if there is matter in Heaven, its condition will be celestial, not terrestrial. On the other hand, it is just as hard to imagine pure mathematics as not surviving. The laws of thought, and especially of number, must hold good in heaven, whether it is a place or a state of mind; for they are independent of any particular sphere of existence, essential to being itself, to God's being as well as ours, laws of His mind

before we learned them. The multiplication table will hold good in Heaven, and on the question of whether twice two make four, we have the mind of Christ. . . . .

Therefore analysis and geometry are the most sacred of all sacred studies; and they are indeed profane persons who pursue them only for examination purposes.

Because pure mathematics is thus a direct contact with God, it is possible for a Christian to dedicate his whole time to research in it; the usual gibe about its usefulness has no meaning. Other professions, and the arts and crafts of everyday life, are worth pursuing only because of their useful applications; they are indirect approaches to God, through the service of man. But mathematics is a direct approach, and can bring the sense of divine companionship more intensely than anything else; for it is God's thoughts that we think after Him, and with Him. We can practise the presence of God in an algebra class better than in Brother Lawrence's kitchen; and in the utter loneliness of an unfashionable corner of research work, better than on a mountain top. If the whole duty of man is to glorify God and enjoy Him forever, who is in a better way of doing his duty than the mathematician?—G. A. Miller, University of Illinois.

(From *School and Society*, March 7, 1925.)

In *Is Mathematics Essentially a Tool*,[1] Professor K. P. Williams of Indiana University points out the seriousness of the mistake in considering mathematics as merely a tool. This mistake is the source of much of the criticism and dissatisfaction with respect to mathematics which has appeared recently. Such a mistaken attitude seems to originate with those who know little of the science, or from those who have failed to grasp its spirit, and for whom it is merely a tool. Against the petty attacks of the present and past time Professor Williams calls attention to the eulogies of great men both in and out of the science, who have grasped somewhat of its spirit, and extending through twenty centuries. Such articles as this render a great service to the interests of mathematics, and to the best interests of education.

(A. D.)

[1] *School and Society*, Feb. 14, 1925.

Mr. Alfred Davis of the Soldon High School, St. Louis, is chairman of a committee of the Missouri State Teachers Association that is reorganizing the syllabus in Trigonometry.

The Mathematics Section of The New York Society for The Experimental Study of Education is giving considerable attention to General Mathematics as a substitute for First Year Algebra. At the March meeting Associate Superintendent Harold G. Campbell spoke on "Conditions Under Which a Test of the Relative Merits of General Mathematics and Elementary Algebra Could Be Made in New York City High School."

The New York Section of the Association of Mathematics Teachers of the Middle States and Maryland held its Spring Meeting at Hunter College, March 20, 1925. Following a dinner, Professor Tomlinson Fort of Hunter College welcomed the members. Dr. Herbert E. Hawkes, Dean of Columbia College, then spoke on Mathematics and the Individual. Professor Lao. G. Simons of Hunter College is chairman of the Section and Miss Edna E. Kramer of Wadleigh High School is Secretary.

Miss Caroline Pfaff, Mr. Auguste J. Tete, Miss Dora Forno, Miss Caroline Wischan and Miss Winnie D. Daly of New Orleans attended the Cincinnati meeting of the National Council of Teachers of Mathematics.

The Eleventh Annual Meeting of the Kansas Section of the Mathematical Association of America was held at Topeka, February 7, 1925. Prof. J. A. G. Shirk gave a report of the Toronto Meeting, International Mathematical Congress. Miss Thirzā Mossaman discussed The Evolution of our Number System. Mrs. Louise A. MacDonald read a paper on Interplay between English and Mathematics Teaching. Professor O. B. Loewen discussed Books Mathematics Teachers Should Read. A general discussion led by Miss Edna Austin on How Can the Association Raise the Standards of Mathematics Teachers? Miss Edith Steininger gave an exposition of Invariants of Determinants with Linear Elements. Prof. U. G. Mitchell presented a paper on Fermat and Mersenne Numbers. Prof A. E. White reported on Formulae for Finding the Standard Deviation and Coefficient of Correlation. A general discussion on Future Policies of the Section was led by Professor J. J. Wheeler.

# MEMBERS OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Continued from the April number.

The name of Professor Earle R. Hedrick, University of Southern California, was inadvertently omitted from the list of members in California. (Editor).

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